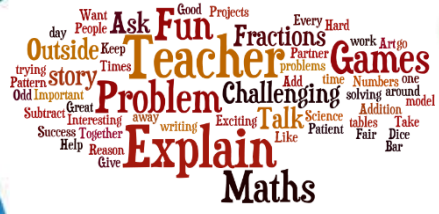


The Russell School

Maths Calculation Policy

The Russell School
Petersham

Updated: February 2016

About Our Calculation Policy

This documents is written for all adults working with our pupils; including teachers, teaching assistants, students, supply teachers and parents. It should be part of an induction package for all staff with inset as appropriate.

Our Calculation Policy has been devised to meet the requirements of the National Curriculum 2014, but most importantly the learning needs of our children at the Russell. The policy has been designed to give pupils a consistent and smooth progression of learning calculations across the school. Teachers should refer to this policy in all planning for calculations including cross curricular links.

The calculation policy is organised according to the requirements that need to be embedded in each year group of the primary curriculum as set out in the National Curriculum 2014; one set of mathematical concepts and big ideas for all. One of our fundamental mathematical Key principles; that this policy has been derived from, is the assumption that children use the language of maths correctly, so that children can develop mathematical concepts and also allows teachers to address misconceptions early and ensure that children have a firm understanding of key mathematical concepts before moving on.

It is vital that children are taught according to the 'stage' that they are working at, the transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time. Throughout this policy stages have been developed which introduces new concepts, outlines appropriate manipulatives and visual models, and what mathematical language is involved for a particular concept. Latter stages are for those children who are showing to have 'mastered' a concept, allowing them to apply their learning in a real life context further deepening their understanding. The new curriculum focuses on skills and mastery and is not about moving children on to the next method as soon as they can do the one before.

Written methods of calculations are based on mental strategies that have been taught using appropriate manipulatives and are only expected once a child has a clear understanding of the processes involved. This policy uses pictorial models that are consistent across year groups which means that skills can be taught, practised and reviewed constantly. These skills lead to more formal written methods of calculation.

Strategies for calculation need to be supported by familiar models and methods to reinforce understanding, such as the whole part model which children are exposed to throughout this policy. The written methods in this document are important but they by no means replace the superb mental methods we have developed. It is important for children to handle manipulatives to develop and reinforce understanding at all stages from Foundation to Year Six. A sound understanding of the number system and the value of a given number (place value) is essential for children to carry out calculations efficiently and accurately. Efficiency in calculation requires having a variety of mental strategies, which are carefully taught at a particular stage in a child's learning. Another key principle is the importance of 10, referred to as 'magic 10' (NCETM, 2015), which allows children to partition numbers to bridge 10, for example 9+6=9+1+5=15, which is helpful to make 10 as this makes this calculation easier.

Children need to be taught and encouraged to communicate their reasoning and thinking at all stages. Confidence in their ability in mathematics and calculations should be encouraged and supported with all children, fostering a 'can do' attitude. The long term aim is for our children to be able to select an efficient method of their own choice asking systematically:

Can I do this in my head?

Can I do this using drawings or jottings?

Do I need to use a pencil and paper procedure?

What resources could I use to help me?

A Malin

January 2016

Year Six

Children in Year Six use their knowledge of the order of operations to carry out calculations involving the four operations and they continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. Pupils begin to extend their knowledge of the number system to include the decimal numbers and fractions that they have met so far and they should understand that there have been different ways to write whole numbers, for example Roman numerals, and that the important concepts of zero and place value were introduced over a period of time.

By the end of Year Six, children should be confident to connect estimation and rounding numbers to the use of measuring instruments and can recall multiplication facts and the related division fluently when solving a given problem, for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6. Children should be confident in using a range of mental and formal methods of recording and apply when solving mathematical problems.

Key Vocabulary: portioned, value, whole part model, negative numbers, decimal, operation, inverse, fact family, addition, subtraction, multiplication, division, array, bar model, compact method, expanded method, exchanging, regrouping, decimal point, whole number, fraction, brackets, laws of distribution, common factor, multiple, prime number, composite number, remainder, percentages, decimal, denominator, numerator.

Key Instant Recall Facts

Autumn 1: I know the multiplication and division facts for all times tables up to 12 x 12.
Autumn 2: I can identify common factors of a pair of numbers.
Spring 1: I can convert between decimals, fractions and percentages.
Spring 2: I can identify prime numbers up to 50.

Number- Number and place value

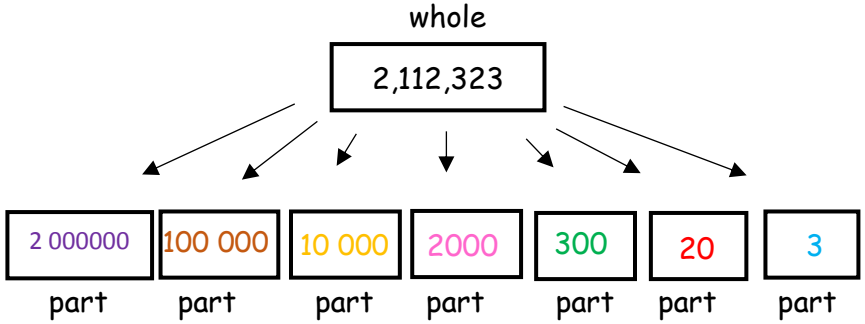
Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit.
Round any whole number to a required degree of accuracy.
Use negative numbers in context, and calculate intervals across zero.
Solve number and practical problems that involve all of the above.

What is the value of the 3rd digit in the number 2,112,323?

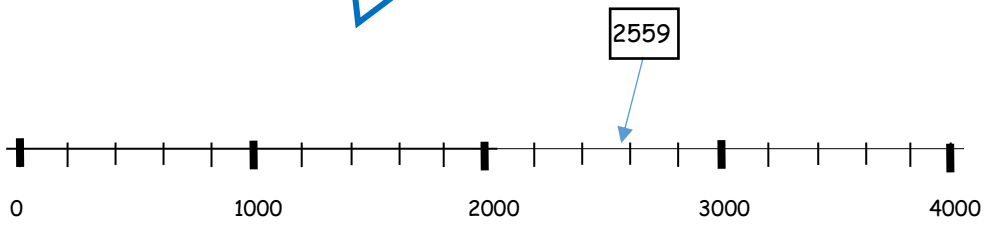
Stage 1- Whole Numbers

Use a partitioning grid alongside place value cards to develop mathematical understanding and language.

Record using whole part model alongside a bar model:



What is 2559 to the nearest 100?



Use a blank number line to continue to develop mathematical understanding of place value when rounding.

Stage 2- Decimal Values

Hundreds	Tens	Ones	Tenths	Hundredths
		3	0	4
0	2	7	5	
	4	3	1	1
5	6	0	3	

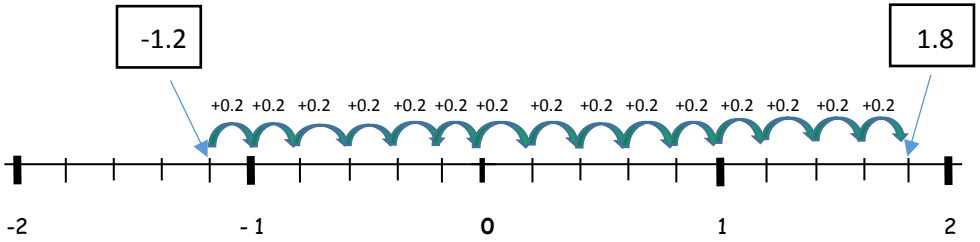
There are 5 **lots of ten** in 50p.

Use a decimal grid with a **fixed decimal point** alongside a place value grid to support mathematical process and continue to develop mathematical understanding of place value in a **context**, such as money or measure.

Stage 3- Negative Numbers

What is the difference between the coolest day and the warmest day of the week?

The difference is 3°C.



Use a decimal grid with a **fixed decimal point** alongside a number line to support mathematical process and continue to develop mathematical understanding of place value in a **context**, such as temperature or measure.

Number- Addition, subtraction, multiplication and division (including decimals)

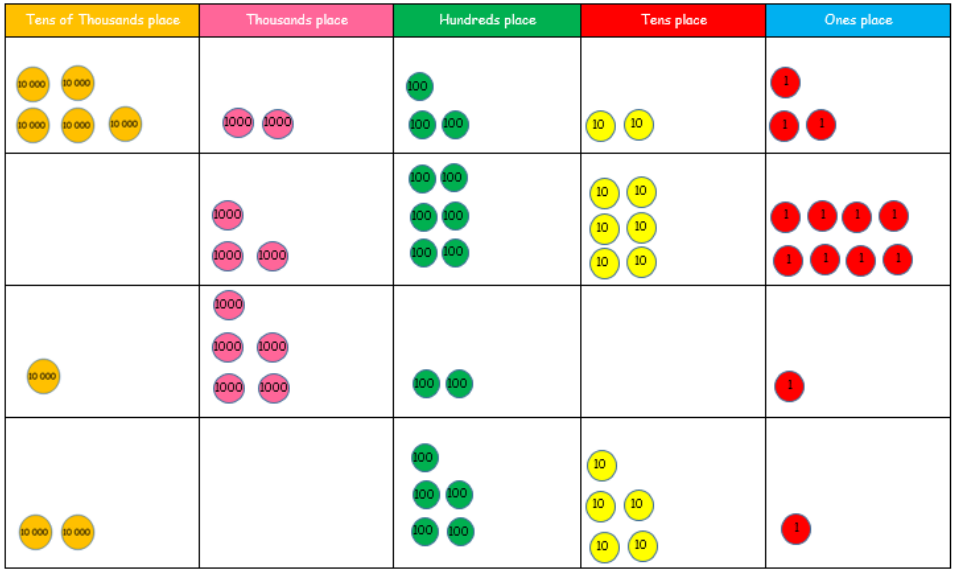
Perform mental calculations, including with mixed operations and large numbers.
Use their knowledge of the order of operations to carry out calculations involving the four operations.
Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why we use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
Use their knowledge of the order of operations to carry out calculations involving the four operations.
Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places

David bought 4 items that cost. They cost £52,323, £3,668, £15,201 and £20,551, how much did he spend altogether?

Addition (The process of finding the total of two or more sets)

Phase 1- Adding several numbers with more than four digits (also in the context of measure, capacity and money)

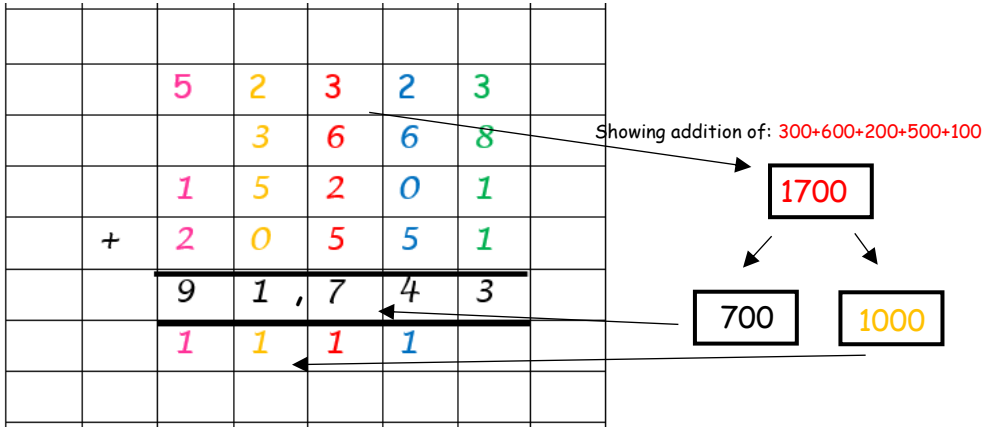
Stage 1



(Place value grid showing: 52,323 + 3,668 + 15,201 + 20,551)

Use place value grid to develop understanding of mathematical process and language.

Record using **compact** column addition, children to use whole part model when partitioning to continue to develop mathematical understanding of place value.



Note: Children should be taught to estimate equation first using rounding to nearest 10/100.

E.g: 52,000+3,700+15,200+20,600= 91,500

Phase 2- Adding several numbers with different numbers of decimal places (also in the context of measure, capacity and money)

Adam measured three pieces of wood, the first piece was 19 cm and 1 mm, the second was 3.65 cm and the third piece was 0.7mm. How long would these pieces be if he joined them altogether?

Stage 1

Hundreds	Tens	Ones	Tenths	Hundredths
	1	9	1	
	3	6	5	
		0	0	7
	5	5	6	7
	1			

Note:

Children should be taught that the decimal point is fixed and that tenths, hundredths and thousandths should be correctly aligned vertically.

Children should also be taught that zeros could be added into empty decimal places, to show that there is no value to add.

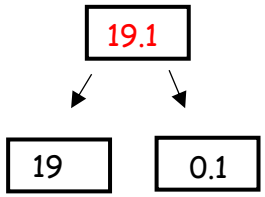
Use a decimal grid with a **fixed decimal point** alongside a place value grid to support mathematical process and continue to develop mathematical understanding of place value.

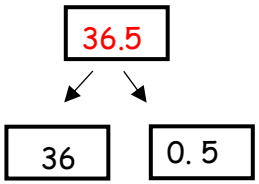
Record using **compact** column addition with a fixed decimal point, children to use whole part model when partitioning to continue to develop mathematical understanding of place value.

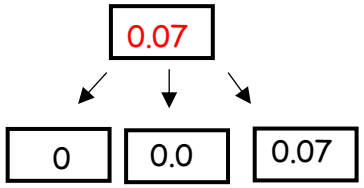
Note: Children should be taught to partition using their knowledge of place value to support mental addition using whole part model.

Firstly, converting all mixed units of measure (cm and mm) to the same unit of measure (cm)

EG: 19 cm 1 mm = 191 mm 3. 65 cm = 36.5 mm 0.7 mm = 0.07 cm







Now, add whole, tenths, hundredths and thousandths parts to find the total.

19.1 + 36.5 + 0.07 = 19 + 36 + 0 = 55
0.1 + 0.5 + 0.0 = 0.6
0.00 + 0.00 + 0.07 = 0.07
= 55 + 0.6 + 0.07 = 55.67

Subtraction- calculating the difference (The process of taking away and counting how many are left)

Phase 1- Subtracting several numbers with more than four digits (also in the context of measure, capacity and money)

David had£150,699 and he spent £89,949. How much money did he have left?

Stage 1

Use place value grid to develop understanding of mathematical process and language.

Record using **compact column** method.

	0	14	9	1			
	1	5	0	6	9	9	
-		8	9	9	4	9	
	0	6	0,	7	5	0	

Stage 2

Develop mathematical reasoning using bar model.

Whole (c)

150,699	
89,949	150,699 - 89,949 = 60,750
Part (a)	part (b)

Record as a fact family.

89,949+60,750=150,699 60,750+89,949=150,699

150,699-89,949=60,750 150,699-60,750=89,949

This can also be generalised as: a+b=c b+a=c c-b=a c+b=a

Phase 2- Subtracting several numbers with different numbers of decimal places (also in the context of measure, capacity and money)

A box had a total mass of 105.419 kg. A man took out 36.08 kg of the objects inside the box. What was the new mass of the box?

Use place value grid with a fixed decimal point to develop understanding of mathematical process and language.

Record using **compact column** method.

	0	19	1		3	1		
	1	0	5	.	4	1	9	kg
-		3	6	.	0	8	0	kg
	0	6	9	.	3	3	9	kg

Note: Empty decimal places can be filled with zero to show the place value in each column.

Children should be encouraged to estimate the answer first using their knowledge of place value to partition each value.

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.

Multiplication

A shop ordered 18 boxes of toys, each box had 13 toys inside. How many toys were there **altogether?**

A shop ordered 25 boxes of toys, each box had 135 toys inside. How many toys were there **altogether?**

Phase 1- Multiply up to a four digit number by a two digit number (also in the context of measure and money)

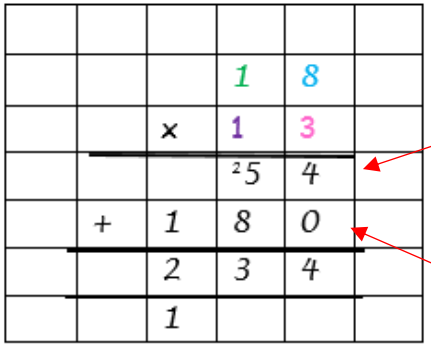
Stage 1- Introducing long multiplication

18 x 13 = 234

x	10	3
10	100	30
8	80	24
	180 +	54

Use grid method alongside place value counters to develop mathematical understanding.

Record using long multiplication method:



			1	8	
		x	1	3	
			²5	4	
	+	1	8	0	
		2	3	4	
		1			

Note: Children to compare grid method to long multiplication method to develop understanding.

18 x 3 on the first row.

(8x3=24, carrying the 2 for twenty, then 1x3).

18 x10 on the second row. Put a zero in the ones column to acknowledge the place holder.

Stage 2- Moving onto more complex numbers.

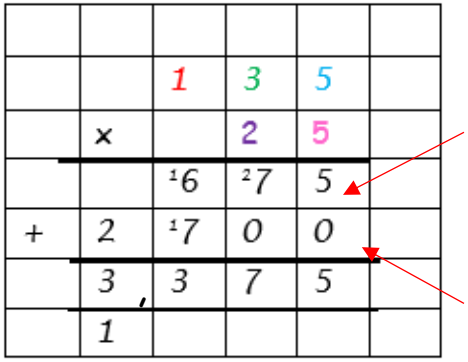
135 x 25 = 3375

x	20	5
100	2000	500
30	600	150
5	100	25
	2700 +	675

Note: Once calculated children can use expanded column addition to find the total.

Use grid method alongside place value counters to develop mathematical understanding.

Record using long multiplication method:



		1	3	5	
	x		2	5	
		¹6	²7	5	
+	2	¹7	0	0	
	3	3	7	5	
	1				

Note: Children to compare grid method to long multiplication method to develop understanding.

135 x 5 on the first row.

(5 x5 = 25, carrying the 2 for twenty, then 5x30=150+20 =170. Carrying the 1 for one hundred, then, 5x100=500+100=600

135 x20 on the second row. Put a zero in the ones column to acknowledge the place holder.

Use a bar model to develop mathematical reasoning.

Multiply one-digit numbers with up to two decimal places by whole numbers.
Use written division methods in cases where the answer has up to two decimal places.

Phase 2- Multiply a one-digit number with up to two decimal places by a whole number (also in the context of measure and money).

David emptied 8 bags of toys into a box, each bag had a mass of 3.19kg. What is the total mass of the box of toys?

		3	.	1	9	Kg
x		8	.			Kg
	2	5	.	5	2	kg
	2	1		7		

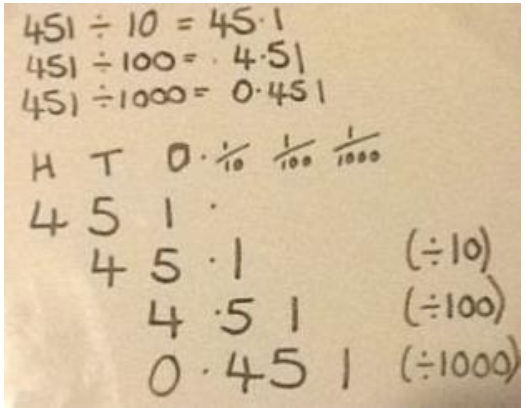
Note: Children should understand that the single digit whole number (8) belongs in the ones column.

Use place value grid to develop understanding of mathematical process and language.

3	.	1	9	X	8	=	2	5	.	5	2		
		0	.	0	9	X	8	=			0	.	7	2
		0	.	1	0	X	8	=			0	.	8	0
		3	.	0	0	X	8	=	+	2	4	.	0	0
										2	5	.	5	2
											1			

Record using **expanded column** method.

Identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places.



Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.
Children are introduced to law of distribution: 'BODMAS'

B	**B**rackets first
O	**O**rders (i.e. Powers and Square Roots, etc.)
DM	**D**ivision and **M**ultiplication (left-to-right)
AS	**A**ddition and **S**ubtraction (left-to-right)

Example: How do you work out **3 + 6 × 2** ?

Multiplication before **A**ddition:

First **6 × 2 = 12**, then **3 + 12 = 15**

Example: How do you work out **(3 + 6) × 2** ?

Brackets first:

First **(3 + 6) = 9**, then **9 × 2 = 18**

Example: How do you work out **12 / 6 × 3 / 2** ?

Multiplication and **D**ivision rank equally, so just go left to right:

First **12 / 6 = 2**, then **2 × 3 = 6**, then **6 / 2 = 3**

Divide and Multiply rank equally (and go left to right).

Add and Subtract rank equally (and go left to right).

After you have done "B" and "O", just go from left to right doing any "D" ***or*** "M" as you find them.

Then go from left to right doing any "A" ***or*** "S" as you find them.

E.g without brackets	E.g with brackets	E.g of developing reasoning skills	E.g of developing problem solving
3 + 4 x 5 = 3 + 20=23 21 - 12÷3=21-4 =17 15 – 6 + 5=9+ 5= 14 7 + 5 – 16 – 9 =-13 7 + 5 – 9 – 16 =-13	(3 + 4) x 5= 7x5=35 (21-12) ÷ 3=9 ÷3=3 15 – (6 +5)=15 -11=4 (7 + 5) –(16 – 9) = 12 – 7= 5 (7 + 5) – (9 – 16) =12-(-7)= 12 + 7= 19	(7 + ☐) –(☐ – 9) = 12 – 7 = 5	Put brackets to make the correct answer: 40 ÷ 5 x 4 = 2 A farmer had 34 animals, 16 of them cows. He sold half the cows at the market and then gave 5 cows to his brother. How many animals does he have now? 34- 16÷2 -5= 21

Identify common factors, common multiples and prime numbers.

When some numbers have the same factor, that factor is called a Common Factor.

Example: Find all the common factors of 12 and 30.

Factors of 12 are 1,2,3,4,6,12.
Factors of 30 are 1,2,3,5,6,10,15,30.

The common factors of 12 and 30 are 1,2,3 and 6.

*When you multiply a given whole number by any other number, the result is a Multiple of that number.

Examples

-The first 3 multiples of 9. Solution: 9x1=9, 9x2= 18, 9x3=27.
-All multiples of 3 greater than 10 but smaller than 20. Solution: 3,6,9,12,15,18,21,24… The desired ones are 12,15,18.

*The common multiples of two numbers are multiples of both numbers.

Example: Common multiples of 4 and 5.
Solution: Multiples of 4 are: 4,8,12,16,20,24,28,32,36,40…
Multiples of 5 are: 5,10,15,20,25,30,35,40….
Common multiples of 4 and 5 are 20,40…

*A Prime Number has exactly 2 factors, the number itself and 1. In other words, the prime number can be divided only by 1 and by itself.
Note: 0 and 1 are not prime numbers.

Examples

-5 is a prime number, because the only factors it has are 1 and 5.
-Find all the prime numbers of 30.
Solution:
All the factors of 30 are 1,2,3,5,6,10,15,30
But only 5,3 and 2 are prime numbers.
So, all prime factors of 30 are 2,3 and 5.

*A Composite Number has at least one more factor than the number itself or 1.

*In fact, all whole numbers that are not prime are composite except for 1 and 0, which are not prime and not composite.

Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate.

Division

Note: When calculating a decimal remainder children should express them as a fraction, decimal or use rounding depending on the problem.

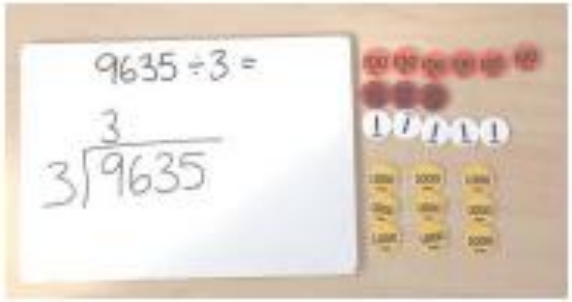
Phase 1- Divide at least a four digit number by a one digit number (also in the context of measure and money).

Use place value counters to develop understanding of mathematical process and language.

Record using **short division** method.

Adam had £9635 and shared it between 3 of his friends. How much money did they each get?

1. Create the dividend using place value counters.



2. Group the 1000s counters according to the divisor and write the number of groups above the line in the thousands column.

Multiple or Dividend

A number that may be divided by another a certain number of times.

Divisor

A number by which another number is to be divided by.

18 ÷ 3 = 6

Quotient

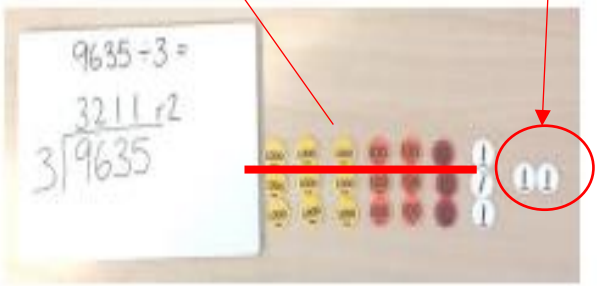
A number that is the result obtained by dividing one quantity by another.

3. Next, group the 100s counters according to the divisor and write the number of groups above the line in the hundreds column.

4. Now, group the 10s counters according to the divisor and write the number of groups above the line in the tens column.

As you look across each group, the quotient can be seen.

Any counters that cannot be grouped are the remainder. Write this at the end as r2.



5. Finally, group the 1s counters according to the divisor and write the number of groups above the line in the ones column. Express the remainders as 'r2' as part of the quotient.

Interpret remainders according to the context (eg money).

Adam had £9635 and shared it between 3 of his friends. How much money did they each get?

Using above example, show remainder as a decimal rounded to the nearest penny.

Each friend got £3211 and 67p each.

Eg: $\frac{9635}{3}$ $(3\overline{)9635}$ = 3211 r2

3	÷	9	6	3	5	=			
		3	2	1	1	.	6	6	6
	3	)9	6	3	5	.	$^{2}0$	$^{2}0$	$^{2}0$

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.

Phase 2- Divide at least a four digit number by a two digit number (also in the context of measure and money).

Adam had £2678 and shared it between 15 of his friends. How much money did they each get?

How many lots of £36 are in £972?

Stage 1- Division using formal method of Long Division

(Dividing by a 2 digit number)

Note: 2 ÷ 15 =0 (Record 0)

26÷15 =1 + remainder (Record 1)

To find remainder 26-15 (1 lot of 15)=11

Now, bring down 7.

117÷15= 7 + remainder (Record 7)

To find remainder 117-105 (7 lots of 15)=12

Now, bring down 8.

128÷15 = 8 + remainder (Record 8)

To find remainder 128-120 (8 lots of 15)=8

2	6	7	8	÷	1	5	=	1	7	8	r8

		0	1	7	8	r8					15
1	5	2	6	7	8						30
	-	1	5	↓							45
		1	1	7							60
	-	1	0	5							75
		0	1	2	8						90
	-		1	2	0						105
			0	0	8						120

Note: Useful list:

Children should make a list of multiples for the divisor (x15 table) to ease calculation.

Eg: 2678 (15)2678 = 178 r8

15

or

178 8/15

Each friend got £178 and 53p.

Stage 2- Introduce Long Division by Chunking

(Dividing by a 2 digit number)

Teach children to subtract chunks of divisor (Eg.36), until zero is reached or until there is a remainder.

How many lots of 36 are in 972?

Using multiplication facts in the useful list: 20 lots of 36 = 720 (Double 10x = 360 = 720)
Remaining 252.

A	w	s	w	e	r	:	£	2	7		

				2	7			1x=	36		
	3	6	9	7	2			2x=	72		
		-	7	2	0	20 x		10x=	360	5x=	180
			2	5	2			100x=	3600	50x=	1800
		-	2	5	2	7 x					
			0	0	0						

How many lots of 36 are in 252?

Using multiplication facts in the useful list: 7 lots of 36 = 252 ((5x = 180) + (2x = 72) =252)
Remaining 0.

Note: Useful list:

Children should make a list of multiplication facts for the divisor (36) to ease calculation.

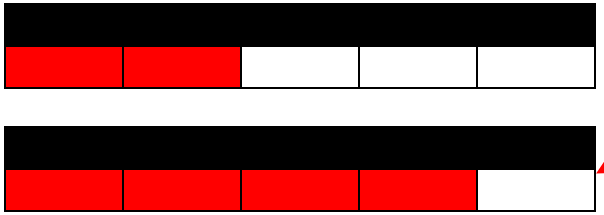
Children to interpret remainders according to the context (eg money) show remainder as a decimal rounded to the nearest penny.

Number – fractions (including decimals and percentages)

Compare and order fractions, including fractions > 1.
Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.
Use common factors to simplify fractions; use common multiples to express fractions in the same denomination.
Multiply simple pairs of proper fractions, writing the answer in its simplest form.

David and Adam made 2 pizzas. David ate $\frac{2}{5}$ of his pizza and Adam ate $\frac{4}{5}$ of his pizza. How much pizza did they eat between them?

Stage 1- Comparing, adding and subtracting fractions that have same denominator



2/5 means that I need to have 5 equal parts and 2 parts are shaded.

Use a bar model alongside an array to support mathematical understanding.
Record as fraction equation.

$$\frac{2}{5} + \frac{4}{5} = \frac{6}{5}$$

Stage 2- Converting improper fractions to mixed numbers

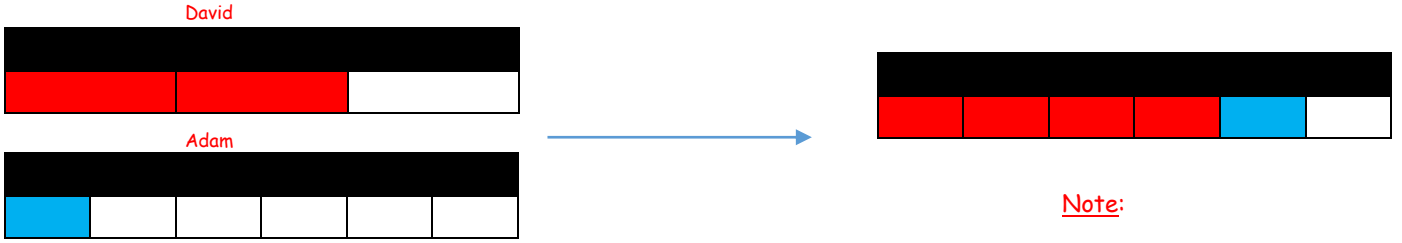
Note: We know that there are two pizzas and ten parts. Between Adam and David they ate 6 parts, hence why 6 parts have been shaded.

Use a bar model alongside an array to support mathematical understanding.
Record as fraction equation.

$$\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$$

Stage 3- Adding or subtracting fractions involving different denominators (Using concept of equivalent fractions)

David and Adam bought a pizzas. David ate 2/3 of the pizza and Adam ate 1/6 of the pizza, how much pizza did they both eat?



Use a bar model to support mathematical understanding.

Record as fraction equation.

$$\frac{2}{3} + \frac{1}{6} \rightarrow \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$$

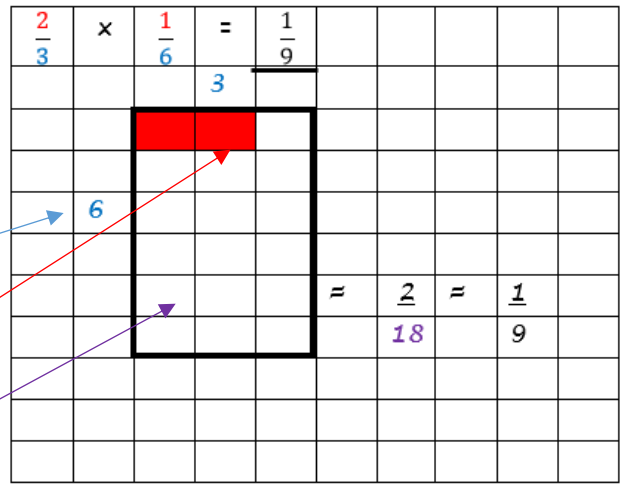
Note:

Find the smallest number, other than 1, which is a factor of both denominators.

(The lowest common multiple of 3 and 6 is 6.
Now multiply $\frac{2}{3} \times 2 = \frac{4}{6}$.)

Multiplying fractions involving different denominators.

What is $\frac{2}{3}$ multiplied by $\frac{1}{6}$?



Note:

Looking at the denominators of both fractions create an array, as if they were whole numbers.

Now, shade in how many parts- looking at the numerator of both fractions.

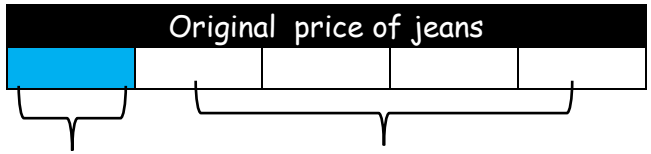
Calculate the whole 6x3 (total area of the array).

Note: Children should be taught to simplify the fraction to its lowest form.

Solve problems which require answers to be rounded to specified degrees of accuracy.
Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.
Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8].
Solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison.

There is 20% off in a sale. The reduced price of the jeans is £36. What was the original price?

I know that 20% is equal to 1/5. So I have divided my bar model into 5 equal parts.



20% £36 (reduced price of jeans)

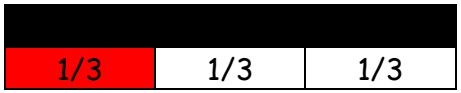
- £36 ÷4= £9
- Each part = £9
- £36+£9=£45

The original price was £45

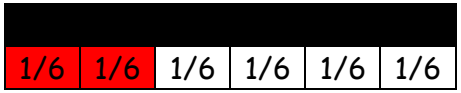
Record using a bar model to support mathematical reasoning.

Divide proper fractions by whole numbers [for example, 1/3 ÷2 = 1/6]

Use a bar model to pictorially represent a problem



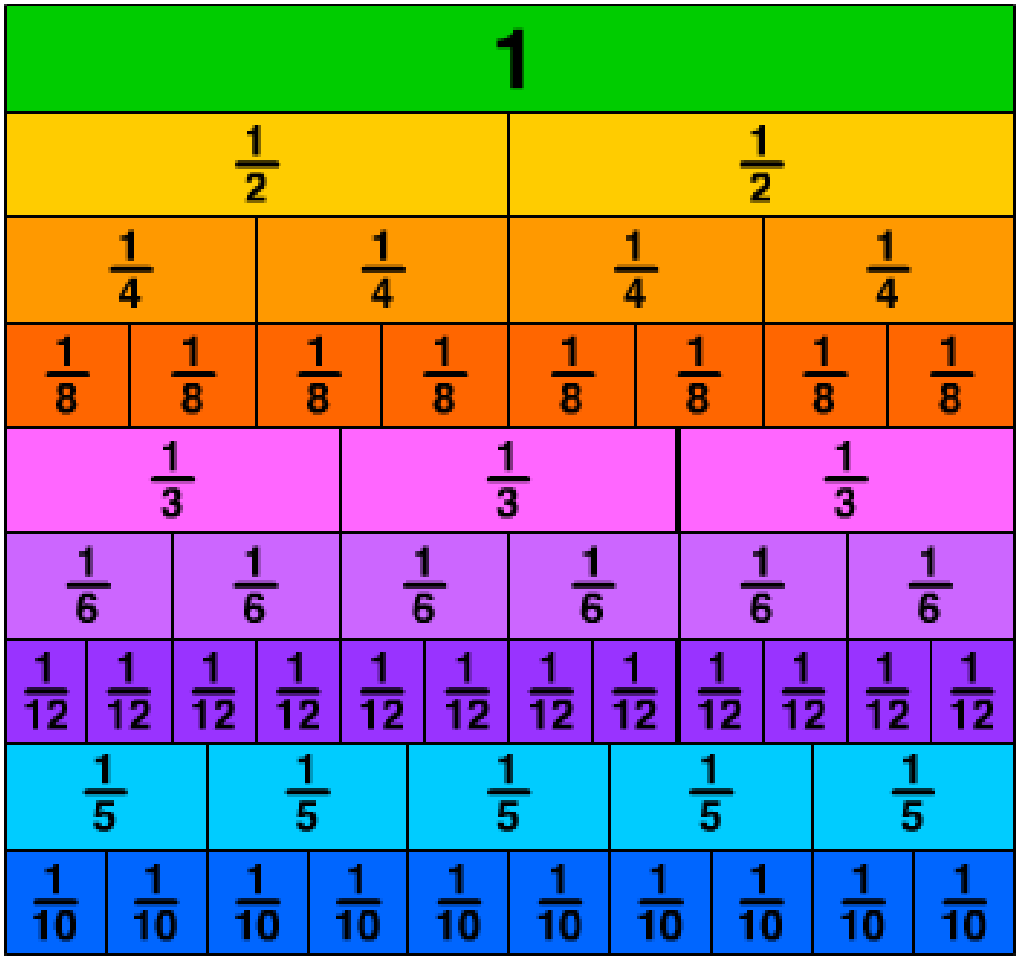
The whole has been divided into 3, each part represents 1/3 of the whole.



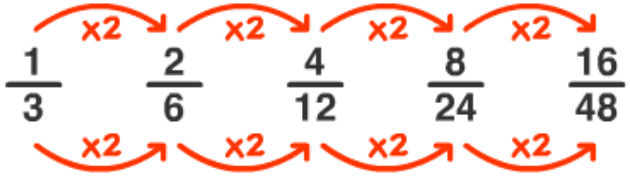
A 1/3 of the bar model has been divided into half, which now represents 1/6 of the whole.

$$\frac{1}{3} \div 2 = \frac{1}{6}$$

Appendix: Fraction Wall visual to be used alongside bar models and fraction manipulatives



You can make equivalent fractions by multiplying or dividing the numerator and denominator by the same number.

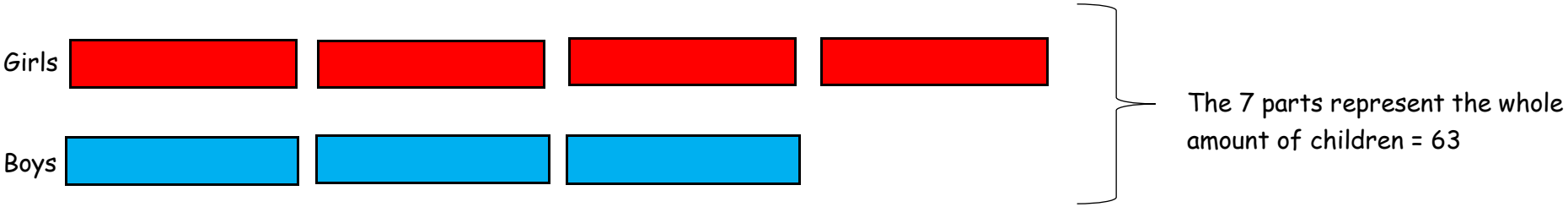


Ratio and proportion

Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.
Solve problems involving similar shapes where the scale factor is known or can be found.
Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

At a dance there are 4 girls to every 3 boys. There are 63 children altogether? How many girls are there?

Use a bar model to pictorially represent a problem



⁂63 ÷7= 9
⁂Each part is worth 9 children.

Girls= 4 lots of 9 (4x9=36)
⁂There are 36 girls at the dance.